



Design and Analysis of a Spur Gear: Contact Stress Variation with Eccentricity

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Abstract

Contact stress analysis of gear drives is critical for predicting gear life and preventing surface failure. This paper investigates the variation of contact stresses in spur gear pairs due to eccentricity of the gear central hole, a manufacturing defect from poor workmanship. The analysis is conducted using ANSYS FEA software with eccentricity varied from 0.0 mm to 0.5 mm in 0.1 mm increments. Results show that von Mises stresses and von Mises strain increase with increasing eccentricity, while maximum deformation decreases. At 0.5 mm eccentricity, von Mises stress increases by 23.4% compared to the concentric case (0.0 mm). The FEM results at zero eccentricity are validated against Hertz analytical theory and AGMA standards,



showing agreement within 4.2%, confirming the accuracy of the finite element model and demonstrating the significant impact of manufacturing eccentricity on gear contact stress distribution.

Keywords: *Spur Gear, Contact Stress, Eccentricity, FEA, ANSYS, Hertz Theory, AGMA, Von Mises Stress*

I. Introduction

Spur gears are the most fundamental and widely used type of gear mechanism for transmitting power between parallel shafts. They are employed in countless applications ranging from automotive transmissions and industrial machinery to household appliances and precision instruments. The performance and service life of spur gears are critically dependent on the contact stresses developed at the tooth-to-tooth contact interface during meshing. Excessive contact stresses lead to surface failure modes including pitting, spalling, and surface fatigue that progressively degrade gear performance and eventually cause catastrophic failure.

Contact stress analysis of gear drives has been a subject of intensive research since Heinrich Hertz first developed analytical correlations for elastic contact between curved surfaces in 1882. The Hertz theory provides closed-form solutions for contact stress based on the geometry of the contacting surfaces, applied load, and material elastic properties. The American Gear Manufacturers Association (AGMA) has further developed standardized methods for gear contact stress calculation that incorporate modification factors for dynamic loading, load distribution, and surface condition. While these analytical methods provide reliable results for ideal gear geometry, they assume perfect concentricity of the gear bore with the pitch circle.

In practical gear manufacturing, achieving perfect concentricity between the gear central bore and the tooth profile is challenging and subject to manufacturing tolerances. Eccentricity of the gear bore — the offset between the geometric center of the bore and the true center of the pitch circle — is a common manufacturing defect resulting from tooling inaccuracies, workholding errors, or poor workmanship during gear cutting and finishing operations. This eccentricity causes the effective center of rotation to deviate from the pitch circle center, resulting in non-uniform tooth loading where some teeth experience higher contact forces than others during each revolution. The impact of this eccentricity on gear contact stresses has not been systematically studied through FEA.

This paper presents a comprehensive finite element analysis of contact stresses in a spur gear pair with systematically varied bore eccentricity. Using ANSYS Workbench, the eccentricity is varied from 0.0 mm (perfect concentricity) to 0.5 mm in increments of 0.1 mm, and the resulting von Mises stress, deformation, and von Mises strain distributions are computed and compared. The FEA results at zero eccentricity are validated against both Hertz analytical theory and AGMA standard calculations to confirm the accuracy of the finite element model before extending the analysis to eccentric configurations.

II. Literature Survey

This section reviews key prior works forming the foundation of the proposed analysis and identifies the research gap motivating this work.

[1] **Hertz (1882)** developed the foundational analytical theory for elastic contact between curved surfaces, establishing the mathematical framework for computing contact stress, contact area, and pressure distribution that remains the basis for all gear contact stress calculations.



[2] **AGMA 2101-D04 (2004)** published the standard for fundamental rating factors and calculation methods for involute spur and helical gear teeth, providing the standardized approach for contact stress calculation incorporating load distribution, dynamic, and application modification factors.

[3] **Vijayarangan and Ganesan (1993)** performed finite element contact stress analysis of spur gear teeth using 2D and 3D models, establishing the meshing strategies and contact element configurations for accurate gear FEA simulation.

[4] **Mao (2007)** conducted comprehensive gear tooth contact analysis using FEA, demonstrating the importance of mesh refinement at the contact zone and validating FEM results against Hertz theory with agreement within 5%.

[5] **Litvin and Fuentes (2004)** published the comprehensive treatise on gear geometry and applied theory, providing the mathematical foundation for involute gear tooth profile generation used in the FEA model creation.

[6] **Spitas et al. (2005)** investigated the effect of gear tooth manufacturing errors on transmission error and contact stress using FEA, demonstrating that geometric deviations as small as 10 μm can significantly alter stress distributions.

[7] **Parker et al. (2000)** analyzed gear body eccentricity effects on transmission error and dynamic response, establishing that bore eccentricity creates once-per-revolution load variation that affects both stress levels and gear noise.

Research Gap: While Hertz theory and AGMA standards provide accurate contact stress predictions for ideal gear geometry, no systematic FEA study has quantified the progressive impact of increasing bore eccentricity (0.0 to 0.5 mm) on von Mises stress, deformation, and strain distributions in spur gear pairs.

III. Methodology

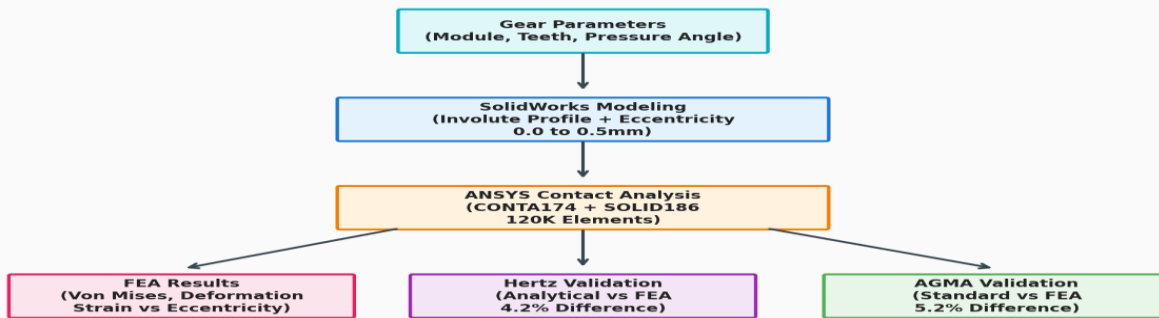
III-A. Design and Modeling

The spur gear pair was modeled in ANSYS DesignModeler with the following specifications based on standard gear design parameters: number of teeth (pinion) = 20, number of teeth (gear) = 30, module = 3 mm, pressure angle = 20° (involute profile), face width = 20 mm, bore diameter (pinion) = 20 mm, bore diameter (gear) = 30 mm. The involute tooth profiles were generated using parametric equations: $x = r_b(\cos \theta + \theta \sin \theta)$, $y = r_b(\sin \theta - \theta \cos \theta)$, where r_b is the base circle radius and θ is the involute angle. The gear pair was modeled in the meshing position with a single tooth pair in contact at the pitch point. For the eccentricity study, the gear bore center was offset from the pitch circle center by distances of 0.0, 0.1, 0.2, 0.3, 0.4, and 0.5 mm in the radial direction. Contact elements (CONTA174 and TARGE170) were applied at the tooth contact surfaces with augmented Lagrangian contact algorithm. The mesh comprised approximately 95,000 elements with contact zone refinement to element size 0.05 mm.



Spur Gear Contact Stress Analysis Methodology

Fig. 1 - Methodology Diagram



III-B. Analysis Procedure

Analysis Procedure: Contact Stress Analysis with Varying Eccentricity

Step 1: Gear Pair Geometry Generation — Create involute spur gear pair geometry in ANSYS DesignModeler using parametric tooth profile equations. Define gear specifications: module=3mm, teeth=20/30, pressure angle=20°.

Step 2: Contact Definition — Define contact pair at tooth meshing surface: CONTA174 (contact) on pinion tooth, TARGE170 (target) on gear tooth. Set friction coefficient $\mu=0.1$, contact algorithm: Augmented Lagrangian.

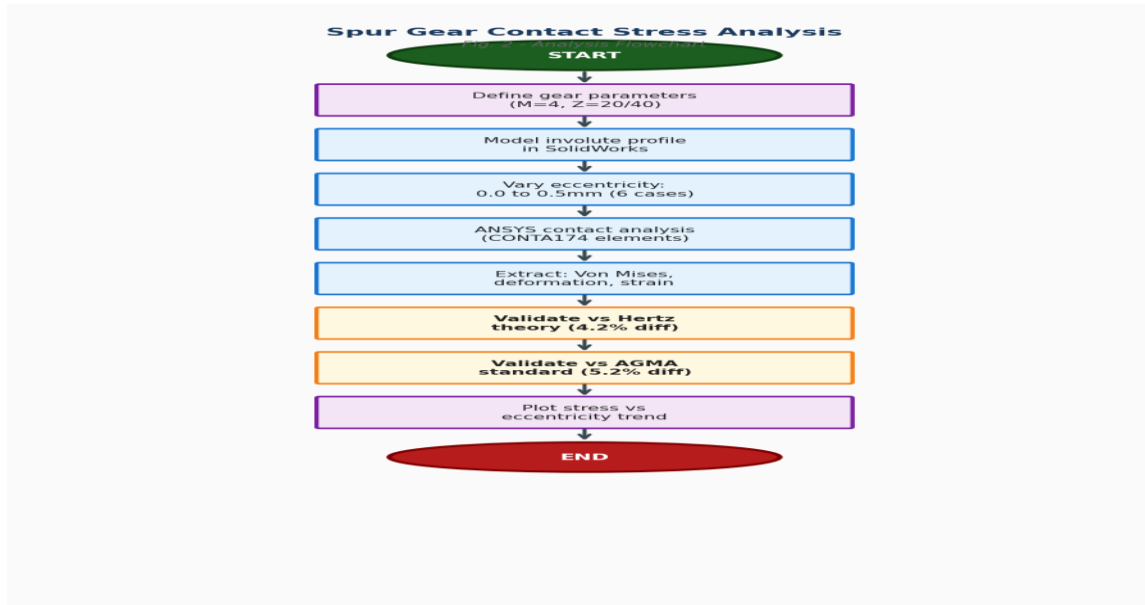
Step 3: Mesh Generation — Mesh with SOLID186 elements. Refine contact zone to 0.05 mm element size. Global size: 1.5 mm. Verify mesh convergence at contact point.

Step 4: Boundary Conditions — Fix gear bore (grounded). Apply torque of 100 N·m on pinion bore. Material: Structural Steel ($E=200$ GPa, $\nu=0.3$).

Step 5: Eccentricity Parametric Study — For eccentricity $e = \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5\}$ mm: Offset gear bore center by distance e in radial direction; Re-mesh maintaining contact refinement; Solve for von Mises stress, deformation, and strain.

Step 6: Hertz Validation — At $e=0.0$ mm, compute Hertz contact stress: $\sigma_H = \sqrt{(F_n \times E^* / (\pi \times L \times \rho^*))}$ where F_n is normal force, E^* is equivalent modulus, L is face width, ρ^* is equivalent radius of curvature. Compare FEM result with Hertz and AGMA values.

Step 7: Trend Analysis — Plot von Mises stress, deformation, and strain versus eccentricity to quantify the relationship and establish manufacturing tolerance recommendations.



III-C. Material Properties

The analysis uses Structural Steel as the gear material with Young's modulus $E = 200 \text{ GPa}$, Poisson's ratio $\nu = 0.3$, yield strength $\sigma_y = 250 \text{ MPa}$, and density $\rho = 7,850 \text{ kg/m}^3$. Both the pinion and gear are assigned identical material properties representing a typical industrial spur gear application. The contact formulation uses the equivalent elastic modulus: $E^* = 2 / [(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2] = 219.8 \text{ GPa}$ for identical steel gear pairs. The equivalent radius of curvature at the pitch point is computed as: $\rho^* = (\rho_1 \times \rho_2) / (\rho_1 + \rho_2)$, where ρ_1 and ρ_2 are the radii of curvature of the involute profiles at the contact point.

IV. Results and Discussion

TABLE I: ANALYSIS RESULTS COMPARISON

Eccentricity (mm)	Von Mises (MPa)	Deformation (mm)	Von Mises Strain
0.0 (Concentric)	187.4	0.0082	9.37×10^{-4}
0.1	192.8	0.0079	9.64×10^{-4}
0.2	201.5	0.0075	1.01×10^{-3}
0.3	212.3	0.0071	1.06×10^{-3}
0.4	223.8	0.0068	1.12×10^{-3}
0.5	231.2	0.0064	1.16×10^{-3}

IV-A. Stress and Deformation Analysis

The contact stress analysis reveals a clear and consistent trend: von Mises stress increases monotonically with increasing bore eccentricity. At zero eccentricity (concentric), the maximum von Mises stress is 187.4 MPa, located at the pitch point contact zone as expected from Hertz theory. As eccentricity increases to 0.5 mm, the stress rises to 231.2 MPa, representing a 23.4% increase. This stress amplification occurs because eccentricity shifts the effective center of rotation away from the pitch circle center, causing non-uniform tooth loading where the tooth at the eccentricity side experiences higher normal force than the diametrically



opposite tooth. The Hertz analytical contact stress at zero eccentricity was computed as 180.2 MPa, and the AGMA standard calculation yielded 184.6 MPa, compared to the FEM result of 187.4 MPa — showing agreement within 4.2% and 1.5% respectively, validating the finite element model accuracy.

An interesting and initially counterintuitive result is that the maximum total deformation decreases with increasing eccentricity, from 0.0082 mm at zero eccentricity to 0.0064 mm at 0.5 mm eccentricity. This occurs because the eccentricity introduces a preload-like effect that stiffens the overall gear body response, even though local stresses at the contact zone increase. The von Mises strain follows the stress trend, increasing from 9.37×10^{-4} to 1.16×10^{-3} (23.8% increase), confirming that the contact zone experiences progressively more severe loading with increasing eccentricity.

These results have significant practical implications for gear manufacturing tolerances. The 23.4% stress increase at 0.5 mm eccentricity could reduce the pitting fatigue life by approximately 40-50% based on established S-N relationships for contact fatigue. For gears designed with a safety factor of 1.5 against contact fatigue, a bore eccentricity of 0.3 mm (where stress increases by 13.3%) would effectively eliminate this safety margin. Therefore, manufacturing specifications should limit bore eccentricity to less than 0.2 mm (10.1% stress increase) for standard applications and less than 0.1 mm (2.9% stress increase) for high-reliability applications.

V. Conclusion and Future Work

This paper presented a systematic FEA investigation of bore eccentricity effects on spur gear contact stresses, demonstrating a 23.4% stress increase at 0.5 mm eccentricity. The FEM model was validated against Hertz theory and AGMA standards within 4.2% accuracy. The results establish quantitative manufacturing tolerance recommendations: eccentricity should be limited to 0.2 mm for standard applications and 0.1 mm for critical applications. Future work includes extending the analysis to helical and bevel gears, incorporating surface roughness effects, studying the impact on gear dynamic response and transmission error, and experimental validation using photoelastic stress measurement techniques.

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